

Critical path and resource levelling

ES, EF, LS and LF: early start, early finish, late start, late finish. Weeks are counted from 0 (ES 2 = the end of week 2). Each formula has a worked example; the three model sheets follow. They serve IPMA, PMI and APM exams alike; APM's PMQ asks most how smoothing differs from levelling.

FORMULAS AND KEY RULES

Forward pass: early start and early finish

ES = the largest EF of the predecessors (0 at the start) · EF = ES + duration

An activity starts only when all its predecessors are done, so where arrows meet, take the largest. Counting from day 1 (PMBOK® Guide): $EF = ES + \text{duration} - 1$; the floats are the same.

Example: A 2-week activity follows two that finish at weeks 5 and 6: $ES = \max(5, 6) = 6$; $EF = 6 + 2 = 8$

Backward pass: late finish and late start

LF = the smallest LS of the successors (the end date at the end) · LS = LF – duration

An activity must end before its first successor must start, so where arrows split, take the smallest.

Example: A 2-week activity feeds three with late starts 2, 3 and 6: $LF = \min(2, 3, 6) = 2$; $LS = 2 - 2 = 0$

Total float and the critical path

total float = LS – ES (= LF – EF) · critical path = zero total float

How long an activity can slip before the end date moves. The chain with zero float is also the longest one; paths a little shorter are near-critical.

Example: ES 2, LS 3: $3 - 2 = 1$ week. Paths of $2 + 4 + 2 + 2 = 10$ and $2 + 3 + 2 + 2 = 9$ weeks: critical and near-critical

Free float

free float = the earliest ES of the successors – EF (last activity: end date – EF)

How long it can slip before a successor must start later. Never more than total float; less when activities with float follow each other.

Example: Activity R runs 2–3, S follows it at 3–5, and S's successor starts at 7: free float R $3 - 3 = 0$, S $7 - 5 = 2$. Late finishes 5 and 7 give both 2 weeks of total float

Resource loading, smoothing and levelling

people in a period = Σ people of the activities running in it

Above the limit is an overload. Ask for people for the peak first. Smoothing holds the end date and shifts work inside float (APM: may change the people); levelling holds the limit, least float first, and the date may move.

Example: Activities with 2, 2 and 1 people overlap in one week: $2 + 2 + 1 = 5$ against a limit of 4, so 1 over

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Sources: IPMA ICB4 (2015); Kelley & Walker (1959); PMI, PMBOK® Guide, 8th ed.; PMI, PMP® Examination Content Outline (2026); APM Body of Knowledge, 8th ed.; APM, PMQ handbook (v6, 2026); IPMA International Certification Regulations (ICR) v4.4 (2025)

Key: IPMA = International Project Management Association; ICB4 = IPMA's Individual Competence Baseline; PMI = Project Management Institute; PMBOK® Guide = PMI's body-of-knowledge guide; PMP® = PMI's Project Management Professional; APM = Association for Project Management; PMQ = APM's Project Management Qualification; ICR = IPMA's International Certification Regulations. Unofficial study material for IPMA®, PMI® and APM exams. Not affiliated with, endorsed by or approved by IPMA, PMI or APM, and not an accredited or endorsed training provider. IPMA® and IPMA ICB® are registered trademarks of IPMA in Switzerland and other countries. PMI, PMP, CAPM and PMBOK are registered marks of Project Management Institute, Inc. APM is a trade mark of the Association for Project Management. Other marks belong to their owners. PMP® exam facts are from PMI's public PMP® Examination Content Outline (2026); check the current version on [pmi.org](https://www.pmi.org). APM's qualifications are its own; this series is not an APM Accredited Provider or APM Endorsed. Examples use a fictional city and its projects.

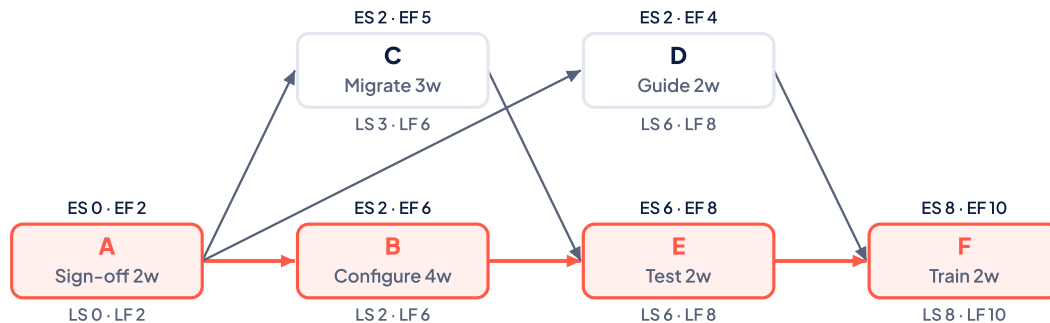


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Critical path method (CPM)

Find the chain that sets the shortest possible duration: the work whose delay moves the end date.



Illustrative, weeks from 0 (2 = end of week 2). Above each box: early start · finish (ES · EF); below: late (LS · LF). Coral: the critical path.

WHAT IT IS

Published by James Kelley and Morgan Walker in 1959. On an **activity-on-node** network, each box is an activity and each arrow a dependency. A forward pass gives each activity's early dates, a backward pass its late ones. Where they agree there is no float: that chain is the **critical path**. It is the longest path, so it sets the shortest possible duration.

HOW IT WORKS

- 01 Forward: ES = the largest EF before it; EF = ES + duration.
- 02 Backward: LF = the smallest LS after it; LS = LF – duration.
- 03 Float = LS – ES; no float: the critical path.
- 04 From day 1: EF = ES + duration – 1; the same float.

MID-LEVEL · WORKED EXAMPLE

Apply it well

The situation: A city team has ten weeks before online parking permits go live: six activities, from signing off the permit rules to training the staff.

Ten weeks on A–B–E–F (2 + 4 + 2 + 2), with no room before go-live; via the data migration, 2 + 3 + 2 + 2 = 9, so near-critical. The project manager books the sign-off now, as A is critical, and checks the migration weekly.

In the exam: Write ES · EF above and LS · LF below each box; name the path, its length and what you will watch.

SENIOR · WORKED EXAMPLE

Design and lead it

The situation: A city is joining 14 online services under one login over 30 months; the national login takes changes only in quarterly release windows.

The project manager sets the rules: one network across the teams, the release windows as fixed dates, and a monthly board view of the critical path and every chain within two weeks of it.

In the exam: Show that you **govern** it: one network, fixed external dates, near-critical chains with owners.

WHERE IT SITS

IPMA · ICB4	PMI · PMBOK® GUIDE 8 · PMP® EXAM	APM · BOK 8 · PMQ EXAM
4.5.4.4: calculate the critical path (p. 114); network analysis, critical path planning (p. 112)	Guide pp. 160–161: critical path method; pp. 185–186: precedence diagramming. Exam outline: Process task 8	BoK (APM's Body of Knowledge) \$5.5.1: critical path analysis (CPA), understood well enough to check software. PMQ (APM's Project Management Qualification) 20b, 20c

Sources: Kelley & Walker (1959); IPMA ICB4 (2015); PMI, PMBOK® Guide, 8th ed.; PMI, PMP® Examination Content Outline (2026); APM Body of Knowledge, 8th ed.; APM, PMQ handbook (v6, 2026)

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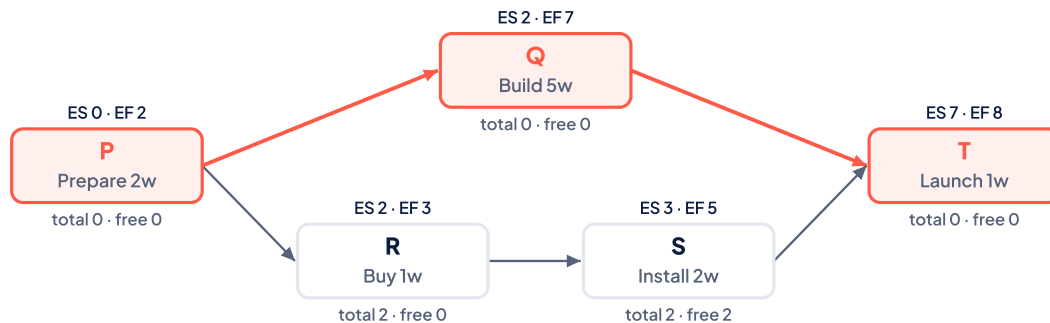
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Total float and free float

How long each activity can slip before the end date moves, or before the next activity must wait.



Illustrative, weeks from 0. R and S share 2 weeks of total float. R has no free float: if it slips a week, S starts later, but the end, week 8, holds.

WHAT IT IS

Float comes out of the two passes of the critical path method (Kelley and Walker). **Total float**: how long an activity can slip without moving the end date. **Free float**: how long before any successor must start later. Free float is never larger. It is smaller when activities with float follow each other, because they share one margin. Some scheduling tools say 'slack'.

HOW IT WORKS

- 01 Total float = late start – early start (LS – ES).
- 02 Free float = next early start – own early finish (EF).
- 03 Two in a row share float: a slip moves the next.
- 04 A date imposed too early gives negative float.

MID-LEVEL · WORKED EXAMPLE

Apply it well

The situation: A city team has ten weeks before online parking permits go live. The data migration has one week of total float; the service-desk guide has four.

Equal here, as each feeds a critical activity. Migration: late start 3 – early start 2 = 1 week, and the test starts at 6, a week after it ends at 5. Guide: 6 – 2 = 4. The migration's week stays a buffer for a known data risk.

In the exam: Give both formulas, then the numbers. Say what they mean: who can wait, who cannot, and what you will watch.

SENIOR · WORKED EXAMPLE

Design and lead it

The situation: A city is joining 14 online services under one login over 30 months, with several teams and a platform supplier.

The project manager sets who may spend float: team leads their free float, which delays no one; total float only with approval, as it takes room from other teams. Supplier milestones come before the city's late dates, so a slip shows early.

In the exam: Treat float as a **shared margin**: who may spend it, how contracts protect it, and who decides when it turns negative.

WHERE IT SITS

IPMA · ICB4

4.5.4: a time buffer or float where timing is uncertain (p. 112); 4.5.4.4 (p. 114)

PMI · PMBOK® GUIDE 8 · PMP® EXAM

Guide p. 161: total, free and negative float; near-critical paths. Exam outline: Process task 8

APM · BOK 8 · PMQ EXAM

BoK (APM's Body of Knowledge) glossary: float, total and free; §5.5.1: critical path analysis finds float. PMQ (APM's Project Management Qualification) 20c

Sources: Kelley & Walker (1959); IPMA ICB4 (2015); PMI, PMBOK® Guide, 8th ed.; PMI, PMP® Examination Content Outline (2026); APM Body of Knowledge, 8th ed.; APM, PMQ handbook (v6, 2026)

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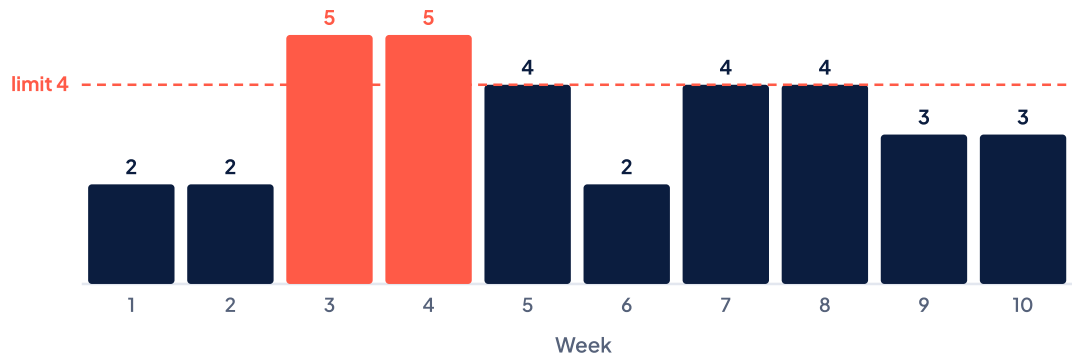
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Resource histogram and levelling

Set the people the plan needs per period against those available, and remove the overloads.



Illustrative early-start plan, people per week; 4 are available. Levelled to 4 (least float first), it takes 11 weeks; still 34 person-weeks.

WHAT IT IS

A resource histogram shows the demand for one resource per period against a line for what is available. Bars above the line are **overloads**. **Resource smoothing** holds the end date and shifts work only inside float; APM (Association for Project Management) also allows changing the people. **Resource levelling** keeps the limit and lets the date move.

HOW IT WORKS

- 01 Add the people of all activities running each week.
- 02 Draw the bars and the limit; mark the overloads.
- 03 Ask for people for the peak first; then smooth.
- 04 Still over? Level, least float first; recheck the path.

MID-LEVEL · WORKED EXAMPLE

Apply it well

The situation: A city team has ten weeks and four people before online parking permits go live. In weeks 3 and 4, three activities overlap and need five.

The project manager first asks for a fifth person for weeks 3 and 4. Smoothing cannot help: no move within float clears the peak. Levelled to four, go-live moves a week, so the steering group chooses: a person or a week.

In the exam: Name the overloaded weeks, give the **first move** (people for the peak), then smooth or level and give the date.

SENIOR · WORKED EXAMPLE

Design and lead it

The situation: A city is joining 14 online services under one login over 30 months. Six integration specialists serve all its projects; a portfolio meeting allocates them.

The meeting allocates three; the plan needs four in months 17–19: $(4 - 3) \times 3 = 3$ specialist-months. Buy them at about €16,000 each ($3 \times €16,000 = €48,000$), borrow one, or level. €48,000 exceeds the board's €40,000 per-decision limit, so it decides.

In the exam: Use it as **portfolio evidence**: demand against allocation, options with cost and effect, and who decides.

WHERE IT SITS

IPMA · ICB4

4.5.4.4: level the resources of a given plan (p. 114); 4.5.8.4: resource conflicts (p. 130)

PMI · PMBOK® GUIDE 8 · PMP® EXAM

Guide pp. 193–194: leveling may move the date, smoothing stays in float; histogram p. 138. Exam: Process task 4

APM · BOK 8 · PMQ EXAM

BoK (APM's Body of Knowledge) §5.6.3, glossary: levelling resource-limited; smoothing time-limited, may change resources. PMQ (APM's Project Management Qualification) 21d

Sources: IPMA ICB4 (2015); PMI, PMBOK® Guide, 8th ed.; PMI, PMP® Examination Content Outline (2026); APM Body of Knowledge, 8th ed.; APM, PMQ handbook (v6, 2026)

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